5.7 Translating English Sentences into Mathematical Equations and Solving

Mathematical equations can be used to describe many situations in the real world. To do this, we must learn how to translate given information into an algebraic equation. Although no single method will work for solving all applied problems, the following approach is suggested to help in the problem-solving process.

KEY WORDS AND PHRASES				
Addition	**Subtraction**	**Multiplication**	**Division**	**Equals**
sum increased by more than plus total combined added to in all	difference decreased by less than minus fewer than reduced by take away	product times of factor of doubled tripled etc.	quotient divided by ratio of per	is/are was/were will be gives yields

STEPS TO SOLVING APPLIED PROBLEMS
Step 1: Read the problem carefully, more than once if necessary, until you understand it. Draw a picture, if necessary. Identify what you are being asked to find.
Step 2: Choose a variable to represent an unknown quantity.
Step 3: Translate the problem into an equation using a well-chosen variable.
Step 4: Solve the equation.
Step 5: Check the answer in the original problem, and interpret the solution as it relates to the problem. Be sure that your answer makes sense in the context of the problem.

Example 1:   Write the following statement as an equation, and determine the number.

Twelve more than a number is seventeen.

Solution

Step 1: Read the problem carefully. We must find an unknown number.

Step 2: Choose a variable to represent the unknown.

Let x = the number.

Step 3: Translate the information into an algebraic equation by rereading the problem slowly and "in parts."

Statement:	Twelve more than	a number	Is	seventeen.
Meaning:	add 12 to	the unknown	Equals	17.
Equation:	+ 12	x	=	17.

The equation is $x + 12 = 17$.

Step 4: Solve the equation.

$$x + 12 = 17$$
$$x + 12 \textcolor{red}{-12} = 17 \textcolor{red}{-12} \qquad \text{Subtract 12 from both sides of the equation.}$$
$$x = 5 \qquad\qquad\qquad \text{Simplify.}$$

Step 5: Check the answer. Does the answer make sense? 12 more than 5 is 17? Yes, the answer is correct.

Practice 1: Write the following statement as an equation, and determine the number. *Four more than a number is negative eighteen.*	
Watch It: http://youtu.be/yZne7g0VVt8	**Answer:** $4 + n = -18$, $n = -22$

<u>Example 2:</u> Write the following statement as an equation, and determine the number.

The difference of a number and 45 is 12.

Solution

Step 1: Read the problem carefully. We must find an unknown number.

Step 2: Choose a variable to represent the unknown.

$$\text{Let } x = \text{the number.}$$

Step 3: Translate the information into an algebraic equation by rereading the problem slowly and "in parts."

Statement:	Difference of a number	and 45	is	12.
Meaning:	x -	45	equals	12
Equation:	x -	45	=	12

The equation is $x - 45 = 12$.

Step 4: Solve the equation.

$$x - 45 = 12$$
$$x - 45 + 45 = 12 + 45 \qquad \text{Add 45 to both sides of the equation.}$$
$$x = 57 \qquad \text{Simplify.}$$

Step 5: Check the answer. Does the answer make sense? Is the difference between 57 and 45 equal to 12? Yes. Therefore x is 57.

<u>Practice 2:</u> Write the following statement as an equation, and determine the number.
The difference of a number and 23 is 4.

<u>Watch It:</u>   http://youtu.be/xL4WX1sqAes   **<u>Answer:</u>** $n - 23 = 4, n = 27$

Example 3:   Write the following statement as an equation, and determine the number.

Five less than two times a number is the same as seven.

Solution

Step 1: Read the problem carefully. We must find an unknown number.

Step 2: Choose a variable to represent the unknown.

Let $x =$ the number.

Step 3: Translate the information into an algebraic equation by rereading the problem slowly and "in parts."

Statement:	Five less than	two times a number	is the same as	seven.
Meaning:	Subtract 5 from	2 times the unknown	equals	7
Equation:	-5	$2x$	=	7

The equation is $2x - 5 = 7$.

Step 4: Solve the equation

$$2x - 5 = 7$$
$$2x - 5 + 5 = 7 + 5 \qquad \text{Add 5 to both sides of the equation.}$$
$$2x = 12 \qquad \text{Simplify.}$$
$$\frac{2x}{2} = \frac{12}{2} \qquad \text{Divide both sides by 2.}$$
$$x = 6 \qquad \text{Simplify.}$$

Step 5: Check the answer. Does the answer make sense? Five less than two times 6 is $2(6) - 5 = 7$. Thus, the answer is correct. The number is 6.

Practice 3:   Write the following statement as an equation, and determine the number.
Ten less than three times a number is the same as eleven.

Watch It: http://youtu.be/AOIfRsq1p9Q   **Answer:** $3n - 10 = 11$, $n = 7$

<u>Example 4:</u> Write the following statement as an equation, and determine the number.

Two fifths of a number is -2.

Solution

Step 1: Read the problem carefully. We must find an unknown number.

Step 2: Choose a variable to represent the unknown.

Let x = the number.

Step 3: Translate the information into an algebraic equation by rereading the problem slowly and "in parts."

Statement:	Two fifths of a number	is	-2.
Meaning:	$\frac{2}{5}$ times x	equals	-2
Equation:	$\frac{2}{5}x$	=	-2

The equation is $\frac{2}{5}x = -2$

Step 4: Solve the equation.

$$\frac{2}{5}x = -2$$

$$\mathbf{5} \cdot \frac{2}{5}x = -2 \cdot \mathbf{5} \qquad \text{Multiply both sides of the equation by 5.}$$

$$2x = -10 \qquad \text{Simplify.}$$

$$\frac{2x}{\mathbf{2}} = \frac{-10}{\mathbf{2}} \qquad \text{Divide both sides by 2.}$$

$$x = -5$$

Step 5: Check the answer. Does the answer make sense? Is two fifths of -5 equal to -2. Yes.

<u>Practice 4:</u> Write the following statement as an equation, and determine the number.
Three fourths of a number is -6.

<u>Watch It:</u> http://youtu.be/7t41S3F1o6U   **<u>Answer:</u>** $\frac{3}{4}n = -6$, $n = -8$

<u>Watch All:</u> http://youtu.be/AmxUlGHU2_c

5.7 Translation Exercises

Translate each of the following into an equation, and then solve the equation.
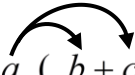
1. The sum of number and 12 is 30. Determine the number.

2. The sum of a number and 2 is 12. Determine the number.

3. The difference of a number and 12 is 30. Determine the number.

4. If 2 is subtracted from a number, the result is 4. Determine the number.

5. If three times a number is increased by 4, the result is -8. Determine the number.

6. When six is subtracted from five times a number, the result is 9. Determine the number.

7. The sum of 8 and 5 is equal to the difference of number and 7. Determine the number.

8. The sum of three times a number and 4 is 19. Determine the number.

9. The sum of twice a number and 5 is eleven. Determine the number.

10. Five times a number decreased by six is 29. Determine the number.

11. If 5 is added to the sum of twice a number and three times the number, the result is 25. Determine the number.

12. Five less than 2 times a number is 7. Determine the number.

13. One half of a number is 24. Determine the number.

14. A number is one tenth less than 1.54. Determine the number.

15. Three tenths of a number is 2.1. Determine the number

16. The quotient of a number and 3 is 10. Determine the number.

17. The difference of 2 times a number and $\frac{3}{4}$ is $\frac{1}{2}$. Determine the number.

18. The product of a number and 2.4 is 0.48. Determine the number.

19. Five eighths added to a number is 4. Determine the number.

20. The product of 5.3 and 10.2 is equal to a number times 0.6.

5.7 Translation Exercises Answers

1. Translation: $x+12=30$

    Answer: $x=18$

2. Translation: $x+2=12$

    Answer: $x=10$

3. Translation: $x-12=30$

    Answer: $x=42$

4. Translation: $x-2=4$

    Answer: $x=6$

5. Translation: $3x+4=-8$

    Answer: $x=-4$

6. Translation: $5x-6=9$

    Answer: $x=3$

7. Translation: $8+5=x-7$

    Answer: $x=20$

8. Translation: $3x+4=19$

    Answer: $x=5$

9. Translation: $2x+5=11$

    Answer: $x=3$

10. Translation: $5x-6=29$

    Answer: $x=7$

11. Translation: $5+(2x+3x)=25$

    Answer: $x=4$

12. Translation: $2x-5=7$

    Answer: $x=6$

13. Translation: $\dfrac{1}{2}x=24$

    Answer: $x=48$

14. Translation: $x=1.54-0.1$

    Answer: $x=1.44$

15. Translation: $0.3x=2.1$

    Answer: $x=7$

16. Translation: $\dfrac{x}{3}=10$

    Answer: $x=30$

17. Translation: $2x-\dfrac{3}{4}=\dfrac{1}{2}$

    Answer: $x=\dfrac{5}{8}$

18. Translation: $2.4x=0.48$

    Answer: $x=0.2$

19. Translation: $\dfrac{5}{8}+x=4$

    Answer: $x=\dfrac{27}{8}$

20. Translation: $(5.3)(10.2)=0.6x$

    Answer; $x=90.1$

CHAPTER 5 SUMMARY
Algebra

Section 5.1	Like Terms: terms that have the same variables raised to the same power

Like Terms: $3x$ and $5x$ Unlike Terms: $3x$ and $5y$

Like Terms: $3x^2$ and $5x^2$ Unlike Terms: $3x$ and $5x^2$

Combining Like Terms – add the coefficients and keep the variable part the same

$$\underbrace{4x+5x}_{9x} \; \underbrace{+3y-8y}_{-\;5y}$$

Evaluating Expressions – replace the variable with the given value and perform the arithmetic using the proper order of operations

Evaluate $7x-2y$ if $x=5$ and $y=3$

$$7(5)-2(3)$$
$$35-6$$
$$29$$

Section 5.2

PROPERTY	DESCRIPTION	ADDITION	MULTIPLICATION
Commutative	Reordering does not change the answer.	$a+b=b+a$ $4+2=2+4$	$a\cdot b=b\cdot a$ $2\bullet3=3\bullet2$
Associative	Regrouping does not change the answer.	$(a+b)+c=a+(b+c)$ $(5+8)+1=5+(8+1)$	$(a\cdot b)\cdot c=a\cdot(b\cdot c)$ $(5\bullet2)\bullet3=5\bullet(2\bullet3)$
Identity	The sum of a term and 0 is that *same* term. The product of a factor and 1 is that *same* factor.	$a+0=a$ $6+0=6$ and $0+a=a$ $0+6=6$	$a\cdot1=a$ $4\bullet1=4$ and $1\cdot a=a$ $1\bullet4=4$
Inverse	The sum of a number and its *opposite* is 0. The product of a number and its *reciprocal* is 1.	$a + (-a) = 0$ $7+(-7)=0$	$a\cdot\dfrac{1}{a}=1$ $3\bullet\dfrac{1}{3}=1$
Zero	The *product* of a number and 0 *is 0*.	———	$a \cdot 0 = 0$ $2\bullet0=0$ and $0 \cdot a = 0$ $0\bullet2=0$

Section 5.3

Distributive Property: $a\;(\;b+c\;)\;=\;ab+ac$

$$2\;(\;3x+4\;)\;=\;2(3x)+2(4)\;=\;6x+8$$

Section **5.4**	<u>Determining if a Given Value is a Solution of an Equation</u>
	
	Is $x = 5$ a solution of $x - 12 = -7$? Replace the variable with the given value.
	            $5 - 12 = -7$ Perform the arithmetic.
	              $-7 = -7$ See if both sides are equal. $x = 5$ *is a solution.*
	
	<u>Addition Property of Equality</u> - If the same number is added to both sides of an equation, the two sides remain equal. If $a = b$, then $a + c = b + c$.
	
	<u>Solving Equations</u> – find the value of the variable that makes the equation true
	     How? Use *inverse* operations to "undo" the operation applied to the variable.
	     Correct Answer? Check your answer by substituting it in the original problem.
	
	<u>Solving Equations Using the Addition Property of Equality</u>
	     Solve $x - 8 = 14$ Determine the operation: Subtraction.
	     $x - 8 + 8 = 14 + 8$ Perform the *inverse* operation: Addition (on *both* sides).
	        $x + 0 = 22$ Do the arithmetic.
	         $x = 22$ The solution is $x = 22$.

Section **5.5**	<u>Multiplication Property of Equality</u> - If both sides of an equation are multiplied by the same number, the two sides remain equal.
	If $a = b$,
	then $a \cdot c = b \cdot c$.
	
	<u>Solving Equations Using the Multiplication Property of Equality</u>
	     Solve $8x = 96$ Determine the operation: Multiplication.
	     $\dfrac{8x}{8} = \dfrac{96}{8}$ Perform the *inverse* operation: Division (on *both* sides).
	       $1x = 12$ Do the arithmetic.
	       $x = 12$ The solution is $x = 12$.

Section **5.6**	<u>Solving Equations Using Both the Addition and Multiplication Properties of Equality</u>
	    $5x + 2x + 2 = 8 - 20$ Solve.
	      $7x + 2 = -12$ Combine *like terms* on each side of the equation.
	    $7x + 2 - 2 = -12 - 2$ Get the *variable term* alone on one side of the equation.
	     $\dfrac{7x}{7} = \dfrac{-14}{7}$ Perform the *inverse* operation. (Inverse of Addition is Subtraction)
	               Get the *variable* alone on one side of the equation.
	      $x = -2$ Perform the *inverse* operation. (Inverse of Multiplication is Division)

Section **5.7**	<u>Translating English Sentences into Mathematical Equations</u>
	  The sum of a number and 15 is 21. 1. Read carefully. What are you asked to find?
	      Let x = the number 2. Select a variable to represent the unknown.
	       $x + 15 = 21$ 3. Translate the words into an algebraic equation.
	    $x + 15 - 15 = 21 - 15$ 4. Solve the equation using inverse operations.
	         $x = 6$ 5. Simplify. (Remember to check your answer.)

Chapter Review

Simplify by combining like terms.

1. $7x - 3x - 8 + 2$
2. $4a - 6a - 5b - b$
3. $\dfrac{1}{8}x + \dfrac{1}{3}x + \dfrac{3}{5} - \dfrac{1}{4}$

Evaluate each expression.

4. $2(x+9)$ if $x = -6$
5. $x^3 + 4$ if $x = -4$
6. $2a - 9b$ if $a = -\dfrac{1}{2}$ and $b = \dfrac{2}{3}$

Distribute.

7. $6(8a - 4)$
8. $-3.9(x + 1.2)$
9. $\dfrac{1}{2}\left(-\dfrac{6}{5}x - 40\right)$

Complete using the property given.

10. $8x + \square = 8x$ Identity
11. $3a \cdot 1 = \square$ Inverse

Use the property given to rewrite and simplify the expression

12. $8 + 5x + 3$ Commutative
13. $4 \cdot \left(\dfrac{1}{4} \cdot a\right)$ Associative

Answer yes or no.

14. Is $x = 8$ a solution of the equation $-15 = -24 + x$?

Solve each equation.

15. $a - 4 = -8$
16. $x + 5.9 = 13.7$
17. $x - 1 + 8 = -3 - 2$
18. $y - \dfrac{1}{3} = \dfrac{2}{5}$
19. $4x = -16$
20. $42a = 6$
21. $-0.4x = -4.8$
22. $-x = 10$
23. $\dfrac{2}{5}x = 10$
24. $\dfrac{x}{7.1} = 2.4$
25. $-8x + 2 = -6$
26. $-18.2 = x - 1.4$
27. $2x + 4 - 2 = 14$
28. $7 + 3x - 8x = 2 - 5$
29. $2(3x - 2) - x = 16$
30. $\dfrac{2}{9} = -\dfrac{7}{9} + a$
31. $\dfrac{1}{8}x - \dfrac{3}{4} = 6$

Translate each into a math equation. Then solve the equation.

32. If four times a number is decreased by five, the result is 31. Determine the number.
33. Two more than the product of 3 and a number is -10. Determine the number.
34. The sum of three times a number and 20 is 11. Determine the number.
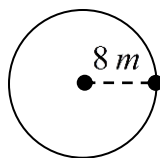35. Seven-tenths of a number is 5.6. Determine the number.

Mixed Review

36. Compute $\sqrt{25} \div \left| -8 - (-3) \right| \times (-14 + 6)$

37. Compute $31.09 + (28.63 - 75.8)$

38. Convert 0.78 g to mg.

39. Convert 12500 seconds to hours.

40. Write 4.06 as a fraction in simplest form.

41. Write $\dfrac{1}{12}$ as a decimal and round to the hundredths place.

42. Compute $4\dfrac{1}{6} - 1\dfrac{3}{8}$

43. Compute $5\dfrac{5}{8} \times 6\dfrac{2}{5}$

44. Compute $\left(\dfrac{2}{3} \right)^2 \div \dfrac{14}{15}$

45. Determine the mean for the following values: 23, –45, 62, –73, –89

46. The players' batting averages are 0.235, 0.312, 0.215, 0.297, 0.310, and 0.233. Determine the median.

47. There are 6 samples of bacteria each weighing $3\dfrac{3}{4}$ grams. What is the total weight of all 6 samples?

48. Determine the perimeter of the rectangle.

$\dfrac{3}{4} cm$

$\dfrac{7}{8} cm$
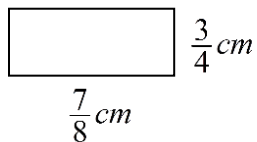
49. Determine the area of the triangle.

4.3 in   3.9 in
2.6 in
8.7 in
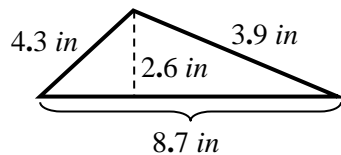
50. Determine the circumference of the circle. Use $\pi = \dfrac{22}{7}$.

8 m

Chapter 5 Review Answers

1. $4x - 6$

2. $-2a - 6b$

3. $\dfrac{11}{24}x + \dfrac{7}{20}$

4. 6

5. -60

6. -7

7. $48a - 24$

8. $-3.9x - 4.68$

9. $-\dfrac{3}{5}x - 20$

10. $8x + \boxed{0} = 8x$

11. $3a \cdot 1 = \boxed{3a}$

12. $5x + 8 + 3 = 5x + 11$

13. $\left(4 \cdot \dfrac{1}{4}\right) \cdot a = 1 \cdot a = a$

14. No

15. $a = -4$

16. $x = 7.8$

17. $x = -12$

18. $y = \dfrac{11}{15}$

19. $x = -4$

20. $a = \dfrac{1}{7}$

21. $x = 12$

22. $x = -10$

23. $x = 25$

24. $x = 17.04$

25. $x = 1$

26. $x = -16.8$

27. $x = 6$

28. $x = 2$

29. $x = 4$

30. $a = 1$

31. $x = 54$

32. $4n - 5 = 31$
    $n = 9$

33. $2 + 3n = -10$
    $n = -4$

34. $3n + 20 = 11$
    $n = -3$

35. $0.7n = 5.6$
    $n = 8$

36. -8

37. -16.08

38. 780 mg

39. $\dfrac{125}{36}\,hrs = 3\dfrac{17}{36}\,hrs$

40. $\dfrac{203}{50} = 4\dfrac{3}{50}$

41. 0.08

42. $2\dfrac{19}{24}$

43. 36

44. $\dfrac{10}{21}$

45. $-24\dfrac{2}{5}$ OR -24.4

46. 0.266

47. $\dfrac{45}{2}\,g = 22\dfrac{1}{2}\,g$

48. $\dfrac{13}{4}\,cm = 3\dfrac{1}{4}\,cm$

49. 11.31 in^2

50. $\dfrac{352}{7}\,m = 50\dfrac{2}{7}\,m$