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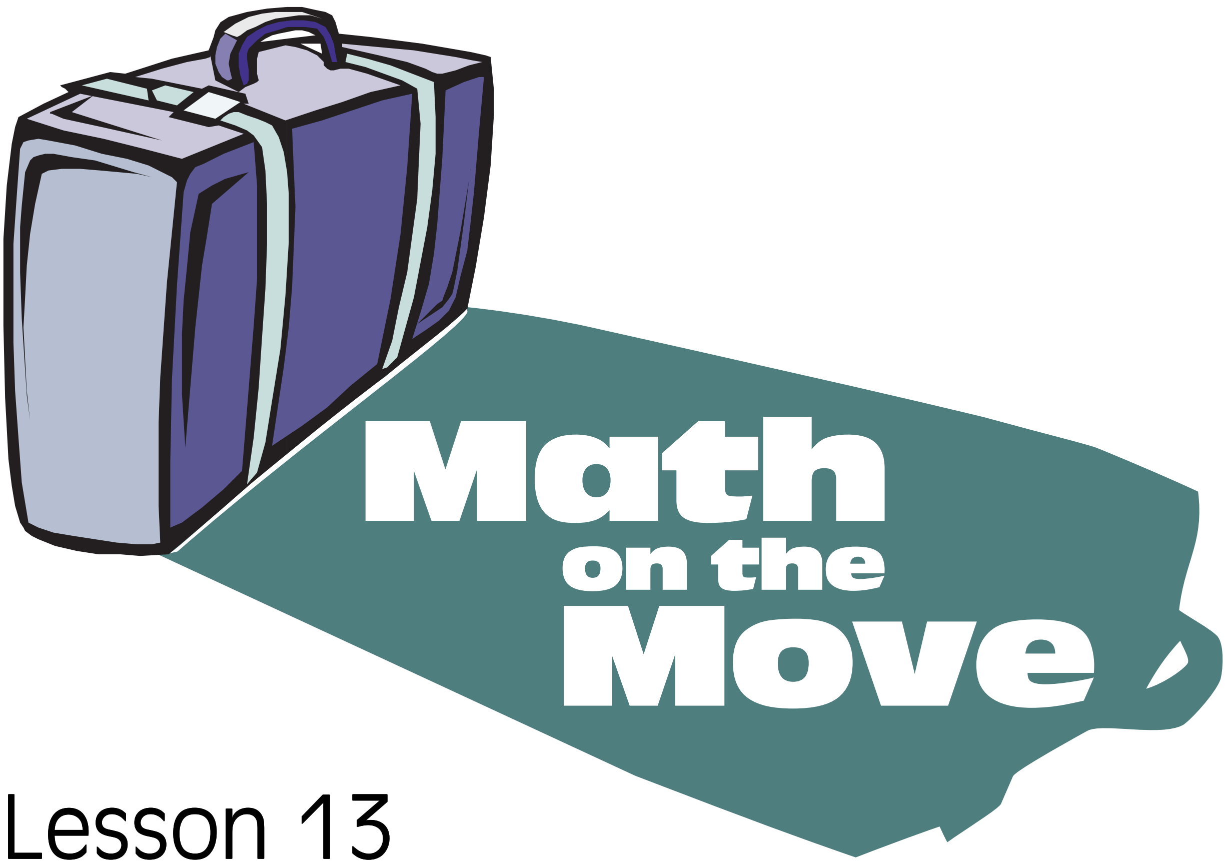

Lesson 13

Verbal Phrases to Algebraic Expressions

Objectives

- Translate verbal phrases into algebraic expressions
- Solve word problems by translating sentences into equations

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One day, you and your family were telling each other different riddles. Your cousin Jesús offers you the following riddle: “Five years ago, I was half the age I will be in eight years. How old am I now?”

In the last two lessons, we discussed variables and how to solve equations for a given variable. In this lesson, we will apply these processes to solve word problems, like the one posed above.

In order to accomplish this, we must determine words and phrases that are commonly used to represent an **algebraic expression**.

- **Algebraic expressions** are made up of constants and variables connected by arithmetic operations. These operations include addition, subtraction, multiplication and division.

The following are examples of algebraic expressions.

Algebraic Expression	Constant(s)	Variable(s)
$3x$	3	x
$5y-6$	5 and 6	y
$\frac{(3s+1)}{4}$	3, 1, and 4	s
$7(4b+2c)$	7, 4, and 2	b and c
$-4x-3y$	−4 and 3	x and y

In each of these algebraic expressions, we see that the constants and the variables are all attached by arithmetic operations. So, we need to find out which phrases are used to stand for different operations. Then, we can represent a verbal phrase as an algebraic expression.

First and foremost, it is important to remember that a variable is used to represent an unknown value. In some cases, a phrase or sentence will tell us which variable we should use to represent the unknown value. However, it is more common for the reader to create the variable using a **let statement**.

- A **let statement** is used to help solve a word problem by creating a variable to represent the unknown value in the word problem.

  For example, "Let *x* = the unknown number."

Now that we know how to write a let statement, we can write one for the word problem at the beginning of the lesson. We do not know Cousin Jesus' age, so we will use a variable to represent this number.

Let *a* = Cousin Jesus' age now

Now that we have created a variable for our word problem, it is time to figure out which phrases mean the different operations.

The following expressions all imply addition.

Let *n* represent the unknown number.

Key Words	Word Expression	Algebraic Expression
plus	6 plus *a number*	$6+n$
added to	*a number* added to 6	$6+n$
increased by	*a number* increased by 6	$n+6$
more than	6 more than *a number*	$n+6$
sum	the sum of 6 and *a number*	$6+n$
total	the total of 6 and *a number*	$6+n$

The Commutative Property of Addition states that 6 plus a number and a number plus 6 represent the same values.

$6 + n = n + 6.$

The following expressions all imply subtraction.

Let *n* represent the unknown number.

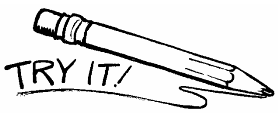
Key Words	Word Expression	Algebraic Expression
minus	5 minus *a number*	$5-n$
	a number minus 5	$n-5$
diminished by	*a number* diminished by 5	$n-5$
	5 diminished by *a number*	$5-n$
decreased by	*a number* decreased by 5	$n-5$
	5 decreased by *a number*	$5-n$
subtracted from*	5 subtracted from *a number*	$n-5$
	a number subtracted from 5	$5-n$
less than*	5 less than *a number*	$n-5$
	a number less than 5	$5-n$

*Notice that with these two key words, the order we subtract the numbers is the reverse of the order we read them in. For example, 10 less than *a number* means n – 10. We write the number 10 second even though it comes first in the phrase.

Think Back

Five minus a number and a number minus 5 are not equivalent. There is no Commutative Property for subtraction since $5-7$ *and* $7-5$ *are not equivalent. Remember that* $5-7=-2$ *and* $7-5=+2$. *Similarly,* $5-n$ *and* $n-5$ *are not equivalent. Expressions must be translated from English to algebraic form exactly.*

It is very important to write the algebraic expression in the correct order. Pay close attention if you see a key word that means subtraction. Try translating the following phrases on your own.



1. Underline the key words in each expression, and then write the algebraic expression implied by each phrase below.

Let n = the number.

Word Expression	Algebraic Expression
a. A number subtracted from 15	
b. 17 more than a number	
c. A number increased by 12	
d. A number decreased by 12	
e. The sum of a number and 8	
f. 15 less than a number	

Let's look at some expressions that imply multiplication.

This time, let *x* represent the unknown number.

Key Words	Word Expression	Algebraic Expression
times	4 times *a number*	$4x$
multiplied by	*a number* multiplied by 4	$4x$
product	the product of 4 and *a number*	$4x$
twice	twice *a number*	$2x$
double	double *a number*	$2x$
triple	triple *a number*	$3x$
of	$\frac{2}{3}$ of *a number*	$\frac{2}{3}x$

FACT

Multiplication is commutative. Remember $3 \cdot 4 = 4 \cdot 3$. *However, when writing algebraic expressions, the constant is always written first. We write* $4x$ *rather than* $x4$. *Therefore a number multiplied by* 4 *is written* $4x$ *instead of* $x4$.

As you can see, there are key words in multiplication that are used for specific numbers. 2 times *a number* can be written as twice *a number* or double *a number*. 3 times *a number* can be written as triple *a number*.

The last operation we need to look at is division.

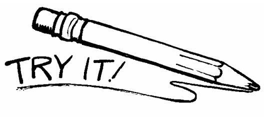
The following expressions all imply division.
Let *a* represent the unknown number this time.

Key Words	Word Expression	Algebraic Expression
Quotient	the quotient of 7 and *a number*	$\frac{7}{a}$
	the quotient of *a number* and 7	$\frac{a}{7}$
divided by	*a number* divided by 7	$\frac{a}{7}$
	7 divided by *a number*	$\frac{7}{a}$

Think Back

Division is not commutative. $\frac{2}{3} \neq \frac{3}{2}$. *Likewise* $\frac{7}{a} \neq \frac{a}{7}$.

Notice that we now longer use the division symbol "$\div$".



2. Underline the key words in each phrase. Then, translate the following expressions into algebraic expressions.

Word Expression	Algebraic Expression
a. four less than a number *b*	
b. six more than a number *r*	
c. the quotient of eleven and a number *t*	

Word Expression	Algebraic Expression
d. three-fifths of a number *y*	
e. a number *z* times eleven	
f. six less than a number *x*	

Now that we have had some practice translating these verbal sentences, let's start relating them to real life situations.

Example

Translate "the number of cents in a given number of quarters."

Solution

One quarter is 25 cents. There are $(25 \cdot 2)$ cents in two quarters.

There are $(25 \cdot 3)$ cents in three quarters.

If we let q equal the unknown number of quarters, then the number of cents in q quarters is $25q$.

TRY IT!

3. Translate the following word expressions into algebraic expressions.

Word Expression	Algebraic Expression
a. the number of cents in *d* dimes	
b. the number of cents in *n* nickels	
c. the number of cents in *x* dollars	

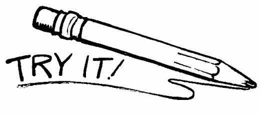
As we learned in Lesson 3, the order of operations is very important. So, when we translate a word expression into an algebraic expression, it is very important to preserve the order of operations. Algebraic expressions must be written and interpreted carefully, so that everyone understands the same meaning. The algebraic expression, $3(x+2)$, is *not* equivalent to the algebraic expression, $3x+2$.

The algebraic expressions on the right represent the word expressions on the left.
The variable *n* will be used to hold the place of the unknown number.

Word Expression	Algebraic Expression
three times a number	$3n$
five more than three times a number	$3n+5$
three times the sum of a number and five	$3(n+5)$

In the second expression, $3n+5$, the number is first multiplied by three; then, that product is added to five. In the third expression, $3(n+5)$, the <u>sum</u> is found first; then, the <u>sum</u> is multiplied by three. The word sum is underlined, because it often means we group the addition in parentheses. The word <u>difference</u> is also important, because it often means we group the subtraction in parentheses.

Try these practice problems on your own.



4. Translate each word expression below into an algebraic expression.

Word Expression	Algebraic Expression
a) the difference of 9 and twice a number *n*	
b) twice the difference of 9 and a number *n*	
c) twice the difference of a number *n* and 9	
d) Rosa's age in *x* years if she is 15 now	

Now that we have worked on translating phrases into algebraic expressions, we need to complete the mathematical sentence in order to create an **algebraic equation**.

- An **algebraic equation** has an algebraic expression set equal to a number or another expression.

  The following are examples of algebraic equations:

  $3x + 2 = 7$ $\qquad$ $3n - 7 = 4n$ $\qquad$ $\frac{a}{5} + 5 = 10a - 8$

The key word for translating complete sentences into an equation is the word "is". All forms of the word "is" represent the equal sign in an equation. Let's look at the word problem at the beginning of this lesson.

"Five years ago, I was half the age I will be in eight years. How old am I now?"

Let a = Jesus' age now

As we can see, the word "is" is not in the word problem, but the word "was" is the past tense of the word "is". So, we will double underline the word "was", because that is where the equals sign goes.

"Five years ago, I was half the age I will be in eight years. How old am I now?"

Let *a* = Jesus' age now

Now we must find the key words that represent operations and numbers. There are no key words in this problem that match the ones given to us already. This problem is written with respect to Jesus' age now.

$a - 5$ $\qquad\qquad$ $\frac{1}{2}$ $\qquad\qquad$ $a + 8$

"Five years ago I was half the age I will be in eight years. How old am I now?"

Let *a* = Jesus' age now

If Jesus is 10 years old now, he would have been 10 – 5 = 5 years old 5 years ago. Since we used *a* to represent his age now, his age 5 years ago would be represented by $a - 5$. Similar to this, his age in 8 years would be $a + 8$. Lastly, the word "half" simply means $\frac{1}{2}$. However, we must multiply his age in 8 years by $\frac{1}{2}$, because the word "half" is the same as saying "half of". So, now we will write this sentence as an algebraic equation.

"Five years ago I was half the age I will be in eight years. How old am I now?"

$$a - 5 = \frac{1}{2}(a + 8)$$

You may be wondering why we put the parentheses around the $a + 8$. If we didn't, it would not mean the same thing.

$\frac{1}{2}(a + 8)$ means that we multiply his age in 8 years by $\frac{1}{2}$

$\frac{1}{2}a + 8$ means we multiply his age now by $\frac{1}{2}$, then add 8 years to that.

So, the parentheses are required to make the statement correct.

Now, we can try to solve the equation.

$$a - 5 = \frac{1}{2}(a + 8)$$

This is the first time we have seen parentheses in an equation. To solve this problem, we must get rid of the parentheses using the **distributive property**.

- Given numbers *a, b,* and *c,* if we have $a(b+c)$, the **distributive property** states that we can multiply "*a*" into the parentheses if we multiply it by everything that is being added inside the parentheses.

$$a(b+c) = ab + ac \qquad 2(k+3) = 2k+6$$

So, we must distribute the $\frac{1}{2}$ into the parentheses.

$$a - 5 = \frac{1}{2}(a + 8)$$

$$a - 5 = \frac{1}{2}a + \frac{1}{2} \cdot \frac{8}{1}$$

$$a - 5 = \frac{1}{2}a + 4$$

Now we can start to solve the equation for *a*.

$$a - 5 = \frac{1}{2}a + 4$$

$$-\frac{1}{2}a \qquad -\frac{1}{2}a$$

$$\frac{1}{2}a - 5 = 4$$

$$+5 \qquad +5$$

$$\left(\frac{2}{1}\right)\frac{1}{2}a = 9\left(\frac{2}{1}\right)$$

$$a = 18$$

According to our work, *a* = 18.

This means that Jesus is 18 years old.

The last thing that we need to do is check our answer using the word problem.

If Jesus is 18 now, he was 13 five years ago, and he will be 26 in eight years.

"Five years ago I was half the age I will be in eight years. How old am I now?"

$$13 \qquad = \qquad \frac{1}{2} \qquad \cdot \qquad 26$$

$$13 = \left(\frac{1}{2}\right)26$$

$$13 = 13$$ ✓

So, we can see that our answer of 18 works. When working with real-life example, evaluate your answer using common sense. If you ended with a negative number for an age, go back and look at your algebraic computations. If you haven't made any mistakes there, you may have translated the algebraic expression incorrectly.

Let's try one more together.

Example

Five more than twice a number is three times the difference of that number and two. What is the number?

Solution

First, we need to create a let statement for our unknown value.

Let n = the number

Next, we will underline all the key words in the sentence. Make sure you double underline the word "is", because that is where the equal sign goes.

Five more than twice a number is three times the difference of that number and two. What is the number?

"More than" means addition. More specifically, "five more than" means we add 5 to something. "Twice a number" is 2 times the variable. So, five more than twice a number is the same as

$$2n + 5.$$

"Three times" means we multiply something by 3. "Difference" means subtraction. More specifically, "the difference of that number and two" means we group the subtraction of the variable and 2. So, three times the difference of that number and two is the same as

$$3(n - 2).$$

Since those two phrases were separated by the word "is", we set them equal to each other.

$$2n + 5 = 3(n - 2)$$

Now we can start to solve the equation.

$$\begin{aligned} 2n + 5 &= 3(n - 2) \\ 2n + 5 &= 3n - 6 \\ -2n \quad &\quad -2n \\ 5 &= n - 6 \\ +6 \quad &\quad +6 \\ 11 &= n \end{aligned}$$

So, the number is 11. Let's check our answer.

Twice the number is 22, and five more than that is 27. The difference of the number and two is 9, and three times that is 27. Since 27 = 27, we know the number 11 is the correct answer.

Notice that when we check our answer, we check it using the written statement instead of the algebraic equation we translated. If we translated our algebraic equation wrong, the answer we get is wrong, but when we check it, it seems right. Therefore, we must check our answer by using the original written statement.

5. Translate the following sentences into equations, and solve.

(Make sure you write a let statement)

a. Twice the sum of four and a number is six less than that same number. What is the number?

b. Three years from now, Alejandra will be triple her age from seven years ago. How old is Alejandra?

When we solve word problems, we often have to write multiple let statements

Example

Three consecutive integers sum to 99. What are the three numbers?

Solution

Before we can start to solve this, we need to understand what **consecutive integers** are.

- **Consecutive integers** are integers that come right after another on the number line.

  The following are examples of consecutive integers.

  {1, 2, 3, 4, 5}   {13, 14, 15, 16}   {-11, -10, -9}

As you can see, consecutive integers increase by one each time. But for our problem, the three consecutive integers are unknown. So, we will use a variable to represent the first number.

Let x = the first integer

If we use x as the first integer, then the second integer should be 1 more than that. Also, the third integer should be 2 more than the first one. So, the following let statements can be used to represent our three numbers.

Let x = the first integer

$x + 1$ = the second integer

$x + 2$ = the third integer

Now we have a variable representation of our three consecutive integers. The next thing it tells us is that the sum of the three numbers is 99. So, we add our three integers and set that equal to 99.

$$x + (x + 1) + (x + 2) = 99$$

We can get rid of the parentheses because there is no number that needs to be distributed through them.

$$x + x + 1 + x + 2 = 99$$

Combine like terms

$$3x + 3 = 99$$

Solve the equation for the variable

$$3x + 3 = 99$$

$$\quad -3 \quad -3$$

$$\frac{3x}{3} = \frac{96}{3}$$

$$x = 32$$

We have found x. But wait! We are not done. The question asked us to find the three numbers, and we only found one number. To find the other numbers, we need to look back at the let statements

Let x = the first integer = 32

$x + 1$ = the second integer = 33

$x + 2$ = the third integer = 34

If $x = 32$ is the first integer, then the second integer is 1 more than that, $x + 1 = 33$. The third integer is 2 more than the first integer $x + 2 = 34$. If we check it,

$$32 + 33 + 34 = 99 \quad \checkmark$$

Let's try one that is a little harder.

Example

Isabela has $2.55 in quarters, nickels, and dimes. She has twice as many nickels as dimes and one less quarter than nickels.

Solution

Our first step is to write the let statements. When writing the let statements, we need to find what our unknown values are. In this problem, we do not know how many quarters, nickels and dimes we have. We first need to figure out which of the coins we know the least about. If we focus on the second sentence of the word problem, we are told how many nickels we have in terms of dimes and how many quarters we have in terms of nickels. But, we are not told how many dimes we have, so we will let the variable d represent the number of dimes we have.

Let d = number of dimes

Since we were told that there are twice as many nickels as dimes, we can write the following let statement.

Let $2d$ = number of nickels

If d is the number of dimes, then $2d$ is two times as many dimes.

Now that we have this let statement, we can write our let statement for quarters. We were told that there is one less quarter than nickels, so we can write the following let statement.

Let $2d - 1$ = number of quarters

If $2d$ is the number of nickels, then $2d - 1$ is one less than the number of nickels.

Now that we have our three let statements, we can translate the word problem into an algebraic equation. To do this problem, we must look back to a problem we did earlier. If all of the coins add up to $2.55, we know that there are 255 cents in that amount of money. So, we need to add up <u>the number of cents</u> in each of our coins, and set it equal to our total number of cents. If there are d dimes, then we have $10d$ cents from our dimes. If there are $2d$ nickels, then we have $5(2d)$ cents from our nickels. If there are $2d - 1$ quarters, then we have $25(2d - 1)$ cents from our quarters. If we add up the total number of cents, it should equal 255.

Let d = number of dimes

$2d$ = number of nickels

$2d - 1$ = number of quarters

$$10d + 5(2d) + 25(2d - 1) = 255$$

The **5**, **10**, and **25** are the values of each coin in cents.

Distribute through the parentheses.

$$10d + 10d + 50d - 25 = 255$$

Combine similar terms. Solve and check.

$$70d - 25 = 255$$
$$\quad +25 \quad +25$$

$$\frac{70d}{70} = \frac{280}{70}$$

$$d = 4$$

d = number of dimes = 4

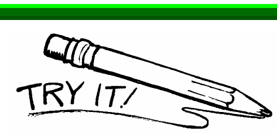
2*d* = number of nickels = 8

2*d* – 1 = number of quarters = 7

Check: 4 dimes, plus 8 nickels, plus 7 quarters is

$$4(10) + 8(5) + 7(25) =$$
$$40 + 40 + 175 = 255 \quad \checkmark$$

Try getting some practice on your own.



6. Solve the following word problems. Then check to see if they are correct.

a. Three consecutive integers sum to 123. What are the three integers?

b. Jada goes to the store and buys two CD's. The CD's come to $18.95, and she gives the cashier $20. She got her change back in quarters and nickels. If she got back twice as many nickels as quarters, how many quarters and nickels did she get back?

Review

1. Highlight the following definitions
    a. algebraic expression
    b. let statement
    c. algebraic equation
    d. distributive property
    e. consecutive integers

2. Highlight the "think back" boxes.

3. Highlight the "fact" boxes.

4. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

Math On the Move Lesson 13

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 13, Set A and Set B.

Set A

1. Translate each word expression into an algebraic expression. Use *b* to represent the unknown number.
    a. seven times a number
    b. nine more than a number
    c. three less than a number
    d. three minus a number

2. Translate the following word expressions into algebraic expressions.
    a. Yolanda's age three years from now if her current age is *a*
    b. four more cents than *q* quarters
    c. the area of *x* tiles if each tile has an area of 144 square inches
    d. eight less than twice a number *n*
    e. six times the sum of a number *n* and 15

Set B

1. What is the difference between an algebraic expression and an algebraic equation?

2. Juan's father is 5 times as old as Juan, and Juan is twice as old as his sister Anita. In two years, the sum of their ages will be 58. How old is Juan, Anita, and their father?

3. Savannah went to the grocery store and bought bananas. The total came to $4.35, and she handed the clerk a 5-dollar bill. The clerk gave Savannah five coins back (in quarters and dimes). How much change did Savannah get back, and how many quarters and dimes did she have? (*Hint*: If x = the number of quarters, what is $5 - x$?)

ANSWERS TO TRY IT

1.	Word Expression	Algebraic Expression
a.	A number subtracted from 15	$15 - n$
b.	17 more than a number	$n + 17$
c.	A number increased by 12	$n + 12$
d.	A number decreased by 12	$n - 12$
e.	The sum of a number and	$n + 8$
f.	15 less than a number	$n - 15$

2.	Word Expression	Algebraic Expression
a.	four less than a number b	$b - 4$
b.	six more than a number r	$r + 6$
c.	the quotient of eleven and a number t	$\frac{11}{t}$
d.	three-fifths of a number y	$\frac{3}{5}y$
e.	a number z times 11	$11z$
f.	six less than a number x	$x - 6$

3. a) $10d$ b) $5n$ c) $100x$

4. Word Expression	Algebraic Expression
a. the difference of 9 and twice a number *n*	$9 - 2n$
b. twice the difference of 9 and a number *n*	$2(9 - n)$
c. twice the difference of a number *n* and 9	$2(n - 9)$
d. Rosa's age in *x* years if she is now 15	$15 + x$

5a. <u>Twice</u> the <u>sum</u> of four and a number <u>is</u> six <u>less than</u> that same number. What is the number?

Let *x* = the number

$$2(4 + x) = x - 6$$

$$8 + 2x = x - 6$$
$$-x \quad -x$$

$$8 + x = -6$$
$$-8 \quad -8$$

$$\boxed{x = -14}$$

b. <u>Three years from now</u>, Alejandra <u>will be</u> <u>triple</u> her age from <u>seven years ago</u>. How old is Alejandra? Let *k* = Alejandra's age

$$k + 3 = 3(k - 7)$$

$$k + 3 = 3k - 21$$
$$-k \quad -k$$

$$3 = 2k - 21$$
$$+21 \quad +21$$

$$\frac{24}{2} = \frac{2k}{2}$$

$$\boxed{12 = k}$$

6a. Three consecutive integers <u>sum</u> to 123. What are the three integers?

Let x = 1st number = 40

$x + 1$ = 2nd number = 41

$x + 2$ = 3rd number = 42

$$x + (x + 1) + (x + 2) = 123$$
$$x + x + 1 + x + 2 = 123$$
$$3x + 3 = 123$$
$$\underline{-3} \quad \underline{-3}$$
$$\frac{3x}{3} = \frac{120}{3}$$
$$x = 40$$

b. Jada goes to the store and buys two CD's. The CD's come to $18.95, and she gives the cashier $20. She got her change back in quarters and nickels. If she got back <u>twice</u> as many nickels as quarters, how many quarters and nickels did she get back?

Let q = quarters = 3

$2q$ = nickels = 6

$$\begin{array}{r} 20.00 \\ -18.95 \\ \hline 1.05 \end{array}$$

25 cents in a quarter

5 cents in a nickel

105 cents in $1.05

$$25q + 5(2q) = 105$$
$$25q + 10q = 105$$
$$\frac{35q}{35} = \frac{105}{35}$$
$$q = 3$$

Jada got back 3 quarters and 6 nickels

End of Lesson 13